THERMODYNAMIC OSCILLATIONS OF HELIUM IN NONUNIFORMLY HEATED TUBES

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Based on an analysis of experimental results, new equations are proposed for describing the thermoinduced oscillations in nonuniformly heated tubes and a mechanical model.

It was observed [1-4] that in the equipment of cryogenic technology and, in particular, of helium technology, oscillations of the working medium may originate spontaneously, sustained by a steady flow of thermal energy. These spontaneous oscillations make the operation of the units difficult and lead to an increase of volatility of the refrigerant, and are considered as extremely undesirable from the point of view of reliable operation of the facility. On the other hand, it is obvious that they can be used for intentional intensification of heat exchange.

Among the different types of thermoinduced oscillations, the self-oscillations of an originally immobile column of helium in nonuniformly heated tubes will be considered here. Similar oscillations were described more than 125 years ago by Zondkhaus [5]. However, up to now they have been little investigated: There is no reliable theory of this phenomenon, no practical methods of calculating these oscillations have been developed, the conditions of their origin are totally unknown, as are reasons for the significant intensification of heat exchange as a result of thermoinduced oscillations. To some extent this is explained by the fact that there is no suitable identification of the experiments, and no models have been constructed which correspond to the experimental results.

We shall consider the possibility of the origination of gas self-oscillations in tubes in the presence of a steady temperature gradient $\partial T_\delta/\partial x$, starting from the acoustic model of laminar oscillations of the gas like the oscillations of an elastic rod. Here the parameters of the system: density $\rho = \rho_0(\rho_\delta + \rho_+)$, temperature $T = T_0(T_\delta + T_+)$, pressure $p = p_0(1 + p_+)$, and velocity v [$\rho_\delta(x)\rho_0$, $T_\delta(x)T_0$ and p_0 are the values of the parameters in the steady state; ρ_+, T_+, and p_+ are their dimensionless perturbations] are connected by the relations which express the local conservation of mass, the local equilibrium of forces, the equation of state and the equation of energy balance, which after linearization with respect to the small parameters p_+, T_+, ρ_+, and v have the form

$$\frac{\partial \rho_+}{\partial t} + \rho_\delta \frac{\partial v}{\partial x} + \frac{\partial \rho_\delta}{\partial x} v = 0, \tag{1}$$

$$V_*^2 \varkappa^{-1} \frac{\partial p_+}{\partial x} + \frac{\partial v}{\partial t} \cdot \rho_\delta + cv = 0, \tag{2}$$

$$p_+ = \rho_+ T_\delta + \rho_\delta T_+, \quad \rho_\delta T_\delta = 1, \tag{3}$$

$$\frac{\partial p_+}{\partial t} = \frac{\varkappa}{\varkappa - 1}\left(\frac{\partial T_+}{\partial t} + v\frac{\partial T_\delta}{\partial x}\right) + \alpha T_+ - \Lambda \frac{\partial^2 T_+}{\partial x^2} \tag{4}$$

with the boundary conditions for a tube, open at one end

$$v(x = 0) = 0$$
$$p(x = L) = 0. \tag{5}$$

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The heat exchange of the gas with the walls of the tube and the axial heat transfer are taken into account by the coefficients $\alpha > 0$ and $\Lambda > 0$. As $\alpha > 0$, $\Lambda > 0$, and usually $\partial^2 T_+/\partial x^2 \backsim L^{-2} T_+$, then the terms αT_+ and $-\Lambda \frac{\partial^2 I+}{\partial x^2}$ in Eq. (4), where $\partial T_+/\partial t$ occurs, play the same role as the frictional resistance cv [1] in Eq. (2), i.e., all these terms correspond to damping of the oscillations of the parameters v, p_+, ρ_+, and T_+. In essence, this follows directly from the first law of thermodynamics and is related with the directional transfer of heat from a warm body to a cold body.

When $c = 0$, $\alpha = 0$, and $\Lambda = 0$, according to relations (1), (2), and (3), both the density and also the pressure and temperature are varying in phase with the movement or acceleration of the gas particles. The steady temperature gradient $\partial T_\delta/\partial x$ and density $\partial \rho_\delta/\partial x$, causing a constant flow of thermal energy, do not change this dependence. Therefore, the effect of these most important terms in the problem being considered is reduced only to a certain change (increase or decrease) of the natural frequency of the acoustic oscillations, which always attenuate when c, α, and $\Lambda > 0$. According to estimates carried out of helium systems, damping of the oscillations by the action of the parameters α and Λ is considerably weaker than by the action of viscous friction. Neglecting the quantities α and Λ in a typical special case, when $T_\delta = 1 - \Theta(x/L - 1/2)$ and $\Theta = \text{const}$, from the system of equations (1)-(4) after simplifications, we obtain

$$\frac{\partial^2 v}{\partial t^2} + c\,\frac{\partial v}{\partial t} = V_*^2\left[\frac{\partial^2 v}{\partial x^2} + \frac{\Theta}{L}\,(\varkappa - 1)\frac{\partial v}{\partial x}\right]. \tag{6}$$

From Eq. (6), with the boundary conditions (5), after exponential substitution of

$$v = a\Omega \exp(\gamma t)\exp\left(\frac{q}{L}\,x\right)\sin\left(\frac{\eta}{L}\,x\right)$$

we obtain

$$(\gamma^2 + c\gamma) + \frac{V_*^2}{L^2}\left[\eta^2 + \frac{\Theta^2(\varkappa - 1)^2}{4}\right] = 0, \tag{7}$$

$$\operatorname{ctg}\eta = \frac{\Theta(\varkappa - 1)}{2\eta}.$$

Whence, $\Omega = \operatorname{Im}\gamma \simeq V_*/L[\eta^2 + \Theta^2(\varkappa - 1)^2/4]^{1/2}$ for $c \ll \Omega$, the damping coefficient δ = 0.5 sec. Consequently, the thermoinduced oscillations observed in practice cannot be described within the scope of the rod model of acoustic oscillations with the boundary conditions (5). In this case, only the natural frequency of the gas–nonuniformly heated tube system is determined, and regardless of the experimental results it is found that the system is asymptotically stable.

Usually, unconsidered end effects exert some influence. If it be supposed that the front of the oscillatory gas motion is continuous and unsmeared, then within the scope of the rod model it is obtained that the maximum compression $p_{+\max}$ and the maximum heating up of the gas $T_{+\max}$ have the profile $(1 + a/L)\cos(0.5\,\pi x/L - 2a)$ on the section $0 < x < L - 2a$, then as the maximum expansion $p_{+\min}$ and the cooling $T_{+\min}$ corresponding to it have the profile $(1 - a/L)\cos(0.5\,\pi x/L)$ on the section $0 < x < L$. As a result, on the section $0 < x < L - 2a$ in the case of gas oscillations, a constant small heating up of the gas and tube occurs, and close to its end on the section $L - 2a < x < L$, cooling occurs. It looks as if thermoinduced oscillations are reversible phenomena when the oscillations of the gas are sustained by a similar constantly existing nonuniform temperature distribution.

It has been established by new experiments on the test rig (Fig. 1), described in [1], that with thermoinduced oscillations, an increase of the displacement amplitude of the helium and a reduction of the pressure amplitude in the resonator sections of the equipment occur, conflicting with the representations of the adiabatic acoustic theory of the oscillations. Thus, e.g., during the thermoinduced oscillations of the pressure amplitude $p_0 p_+ = 3 \cdot 10^2$ N/m², in a half-open tube of length L = 3 m, a displacement amplitude of the helium was observed of $a \approx 0.2$ m, whereas according to the acoustic theory, the displacement amplitude is determined by the relation $a_A = (2L/\varkappa\pi)p_+$ ($p_0 = 10^5$ N/m² is the average pressure in the tube during the period of the oscillations) and for the case being considered is equal to $a_A = 4 \cdot 10^{-3}$ m.

On the photograph (Fig. 2) taken through a transparent slit of the dewar flask, the working tube 2 (Fig. 1) can be seen and the helium ejected from it during the thermoinduced oscillations. These ejections clearly show the oscillation amplitude of the helium. The same amplitudes, but photographed poorly, can be seen also during the reverse path.

As a consequence of the enhanced displacement amplitudes, the thermoinduced oscillations in helium equipment are accompanied by considerable heat preflows to the liquid helium, increasing the volatility of the helium in the system by a factor of 15 in our experiments.

In the conditions being considered the rod analogy clearly is violated during exit of the gas from the tube. Here an intense remixing of the gas "exhaled" from the tube occurs, and a considerable cooling of it occurs by the surrounding medium during the intense mass exchange with it. During the reverse motion of the gas, a part of the intermittently cooled gas enters the tube (Fig. 3), which is accompanied by a pressure drop in the tube and an intensification of the gas intake. Later, in the course of an almost complete period, the normal acoustic oscillations occur, which are damped under the action of friction and heat exchange [the parameters c, α, and Λ in Eq. (1)-(4)]. At the start of the next period, a recurrent intermittent change of temperature takes place again, swinging the oscillations.

These motions of the gas are relaxation oscillations, which are described approximately by the equation

$$\frac{\partial^2 f}{\partial t^2} + c_0 \frac{\partial f}{\partial t} + \Omega_0^2 f = 0,$$

$$f(t = nt_* + 0) = (1 + A) f(t = nt_* - 0) \tag{8}$$

or

$$\frac{f(t = nt_*)}{f[t = (n-1)t_*]} = (1 + A)\exp(-0.5c_0 t_*) \approx (1 + A)(1 - 0.5c_0 t_*), \tag{9}$$

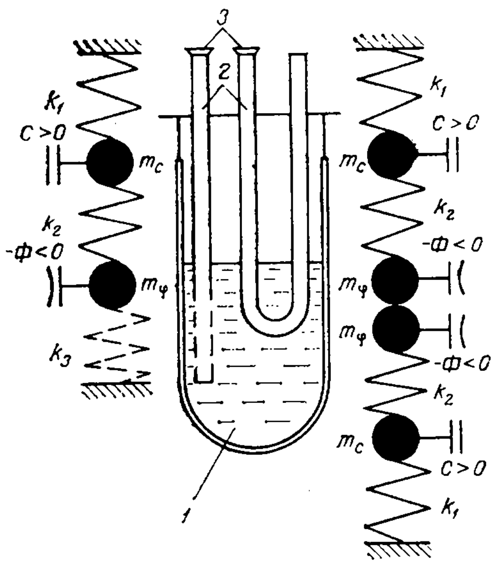

Fig. 1. Diagram of test rig for investigating thermoinduced oscillations and the mechanical model for describing their properties: 1) dewar flask with liquid helium; 2) working tube; 3) pressure sensor.

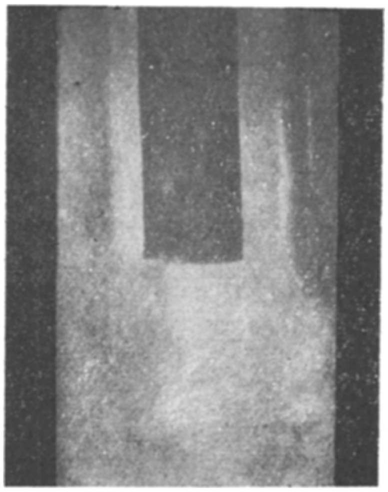

Fig. 2

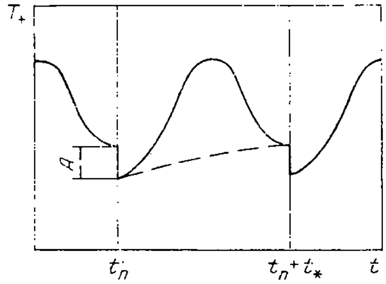

Fig. 3

Fig. 2. Visual observations of thermoinduced oscillations.

Fig. 3. Time-dependent temperature T_+ near an open tube end with gas fluctuations under the conditions of mass transfer with an environment.

where f is the magnitude of p_+ or T_+, or ρ_+, averaged over the length of the tube; c_0 is the generalized coefficient of viscous friction taking account of α and Λ, and $\Omega_0 = \Omega$ when $c \ll \Omega$.

The excitation of the oscillations depends on the degree of mass exchange between the gas emerging from the tube and the surrounding medium, which is expressed by the coefficient ε, $0 \leqslant \varepsilon \leqslant 1$, and on the relative temperature drop $\Delta T T_\varphi^{-1}$, where ΔT is the temperature difference of the gas emerging from the tube and the surrounding medium T_φ. In addition, taking into account that the viscous friction is distributed over the whole length of the tube, and that excitation of the oscillations is directly related with their amplitude, we can write $A \simeq (a/L)\cdot(\Delta T/T_\varphi)\varepsilon$.

The coefficient of mass exchange depends to a significant degree on the local conditions and also on the amplitude of the oscillations. It has been visually observed (Fig. 2) that in the presence of local obstructions around the open end of the tube, reached by the gas wave as a result of the increased amplitudes of the oscillations, the front of the gas column is found to be strongly perturbed and diffuse. The turbulence of the front, in its turn, intensifies both the mass exchange and the heat exchange inside the tube and, as a result, increases the excitation of the oscillations. This is manifested differently on contact of the oscillating gaseous helium column with the liquid helium in the test rig (Fig. 1) with the approach of the open end of the tube to the surface of the liquid helium. The considerable intensification of the thermoinduced self-oscillations observed in this case is explained as an increase of the coefficient of mass exchange ε and also as an increase of the quantity $\Delta T T_\varphi^{-1}$ during spraying of the liquid and an increase of its thawing in the tube 2 (Fig. 1).

It must be assumed that in the different equipment of cryogenic technology or with a different arrangement of the vibroactive tubes in one equipment, the parameters ε and $\Delta T T_\varphi^{-1}$ may have a significantly different but fairly predictable value. Moreover, when the conditions of the experiment are maintained even in different equipment of the same type, these parameters are found to be quite constant.

Taking account of the loss of energy by the involvement in oscillation during mass exchange of part of the gas from the surrounding medium (inlet losses) [6, 7], and in view of the relatively low friction and excitation, the condition of stability of the oscillations (8) is written in the form

$$c\Omega^{-1} + 0{,}6\varepsilon a L^{-1} \geqslant A\pi^{-1} \approx \varepsilon\pi^{-1}\frac{\Delta T}{T_\varphi}\cdot\frac{a}{L},$$

where

$$c\Omega^{-1} \approx \sqrt{2}I^{-1},\quad I = 0{,}5D\sqrt{\Omega/\nu}. \tag{10}$$

Consequently, self-oscillations are excited here only in the presence of a random, not too small pulse, and only with the condition that $\Delta TT_{\varphi}^{-1} > 0.6\pi \approx 2$. In particular, this occurs in the conditions of the test rig of Fig. 1, where $c\Omega^{-1} \sim 10^{-2}$, $\varepsilon \sim 0.7$, $\Delta TT_{\varphi}^{-1} \sim 4$, and $aL^{-1} \sim 2\cdot10^{-2}$.

Under normal conditions, when $c \ll \Omega$ and $A \ll 1$, the oscillations being considered differ slightly from harmonic oscillations and then, in place of the quantity A, it is more convenient to use its equivalent coefficient of negative friction $\varphi\Omega^{-1}$.

In this case, the equation of energy balance is written in the form

$$\frac{\partial p_+}{\partial t} = \frac{\varkappa}{\varkappa - 1}\left(\frac{\partial T_+}{\partial t} + v\,\frac{\partial T_\delta}{\partial x}\right) - \varphi p_+. \tag{11}$$

When solving Eq. (1)-(3) the specific properties of excitation of the oscillations in accordance with the model considered with violation of the rod analogy should be borne in mind. Because of this, only oscillations of the lowest form should be considered, which are characterized by the maximum ejection of gas from the tube into the surrounding space, and the solutions of the higher forms of the system of equations (1)-(3) and (11) should be detached as conflicting with the true excitation. The equation for the gas velocity is obtained from Eq. (1)-(3) and (11)

$$\frac{\partial^3 v}{\partial t^3} + [c - 2(\varkappa - 1)\varphi]\,\frac{\partial^2 v}{\partial t^2} + \left[\varphi^2(\varkappa - 1)^2 - 2c\varphi(\varkappa - 1)\right]\frac{\partial v}{\partial t} =$$
$$= V_*^2\left[\frac{\partial^3 v}{\partial t\partial x^2} + \frac{\Theta}{L}(\varkappa - 1)\frac{\partial^2 v}{\partial t\partial x} + \varphi(\varkappa - 1)\left(-\frac{\partial^2 v}{\partial x^2} + \frac{\Theta}{L}\cdot\frac{\partial v}{\partial x}\right)\right], \tag{12}$$

from which, assuming φ, equal to the quantity $\varphi = \langle\varphi\rangle$ averaged over the coordinate is a constant for small values of Θ and φ, the condition of stability follows:

$$(\varkappa - 1)\varphi < c < \frac{\pi^2\Omega_0^2}{8(\varkappa - 1)\varphi} \quad \text{for} \quad \varphi \ll \Omega. \tag{13}$$

Expression (13) determines the asymptotic boundaries of the region of stability for small and large coefficients of viscous friction c, which are characteristic for many oscillatory processes [6]. The displacement amplitude of the oscillating medium is expressed by the relation

$$a = p_+\frac{2L}{\varkappa\pi}\left[1 + \frac{\varphi(\varkappa - 1)}{\Omega}\right] \tag{14}$$

and is greater, the greater is the quantity φ.

In U-shaped tubes (Fig. 1) an intense excitation of thermoinduced oscillations has also been observed. Here there is no explicit mass exchange with the surrounding space, but it takes place between the gas in the vertical and the condensate in the bent sections of the tube. Moreover, the oscillations here are sustained like the action of a thermal tube with the periodic throwout of condensate into the vertical warmer part of the tube.

For greater visualization it is appropriate to further simplify the oscillatory system being considered by representing it in the form of a two-mass mechanical model (Fig. 1), in which masses m_c and m_φ correspond to the warm and cold parts of the mass of the gas column (helium), participating in the mass-exchange, the elasticities k_1, k_2, and k_3 represent the elasticities of the parts of the gas column, $C_c = C$ and $C_\varphi = -\Phi < 0$ are the coefficients of positive and negative friction of the masses m_c and m_φ.

The stability of this mechanical system is found from the analysis of the equations of motion

$$\begin{gathered} m_c\frac{\partial^2 x_c}{\partial t^2} + C\frac{\partial x_c}{\partial t} + (k_1 + k_2)x_c - k_2 x_\varphi = 0, \\ m_\varphi\frac{\partial^2 x_\varphi}{\partial t^2} - \Phi\frac{\partial x_\varphi}{\partial t} + (k_2 + k_3)x_\varphi - k_2 x_c = 0 \end{gathered} \tag{15}$$

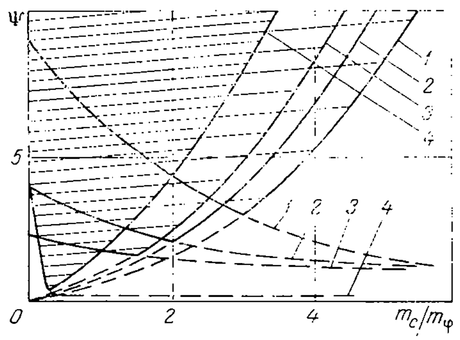

Fig. 4. Dependence of the ratio Ψ of the coefficients of friction and negative friction on the ratio of the masses m_c/m_φ for c_M and $\varphi_M \to 0$; 1) $k_3 = 0$, $k_2 = 0.5k_1$; 2) $k_3 = 0$, $k_2 = k_1$; 3) $k_3 = 0$, $k_2 = 2k_1$; 4) $k_3 = 4k_1$, $k_2 = k_1$. The regions of stability are hatched. The dashed lines are the continuations of the branches bounding the regions of stability.

and according to the Routh–Hurwitz conditions it is satisfied when the inequality

$$F_1F_2\,[m_c\,(k_2+k_3)+m_\varphi\,(k_2+k_1)-C\Phi] > (k_1k_2+k_1k_3+k_2k_3)\,F_1^2+m_cm_\varphi F_2^2. \tag{16}$$

is fulfilled. Here $F_1 = m\varphi C - m_c\Phi$ and $F_2 = (k_2 + k_3)C - (k_2 + k_1)\Phi$; x_c and x_φ are the absolute displacements of the masses m_c and m_φ, respectively.

From condition (16) the asymptotic boundaries of the region of stability for small values of φ_M are expressed by the relation

$$\Psi\varphi_M < c_M < \frac{m_\varphi k_2^2}{m_c^2\,(k_2+k_3)\,\varphi_M} \quad \text{for} \quad \varphi_M \ll \Omega, \tag{17}$$

where

$$\Psi = \Pi + \sqrt{\Pi^2 - \frac{m_c^2}{m_\varphi^2}}\,; \quad c_M = \frac{C}{m_e}\,; \quad \varphi_M = \frac{\Phi}{m_c}\,;$$

$$\Pi = \frac{m_c}{m_\varphi} + \frac{1}{2}\left(\frac{k_1+k_2}{k_2} - \frac{k_2+k_3}{k_2}\cdot\frac{m_c}{m_\varphi}\right)^2.$$

The ratio of the displacement amplitudes a_φ and a_c of the masses m_φ and m_c at the boundary of stability is determined by the expression

$$\frac{a_\varphi}{a_c} = \frac{k_1+k_2-m_c\Omega^2}{k_2}, \tag{18}$$

$$\Omega^2 = B \mp \sqrt{B^2 - \frac{k_1k_2+k_1k_3+k_2k_3}{m_cm_\varphi}}, \quad B = \frac{k_1+k_2}{2m_c} + \frac{k_2+k_3}{2m_\varphi},$$

which is simplified similarly to relation (14). Here, just as in the case of thermoinduced oscillations, the displacement amplitude during oscillations of the system with negative friction is greater than for the natural oscillations, comparable in stresses, in springs (pressure analog).

The system considered can be stable only with a finite value of excitation φ_M, less than the limiting value $\varphi_{M*} \sim k_2/m_c \times \sqrt{m\varphi/k_2 + k_3}\ \Psi^{-1/2}$, to which corresponds the optimum deformation $c_{M*} \sim (k_2/m_c)\ \sqrt{m\varphi/k_2 + k_3}\ \Psi^{1/2}$. The ratio of the quantity φ_{M*} to the actually existing excitation φ_M expresses the margin of stability. The system is unstable for both small values of the coefficient of friction c_M and for large values.

In actual hydrodynamic systems the viscous resistance is small ($c_M \ll \Omega$), the stability in this case is determined by the quantity $\Psi = \varphi c_M/\varphi_M$ for c_M and $\varphi_M \to 0$, the dependence of which on the parameters of the system is represented in Fig. 4, in which the left-hand branch of the region of stability with argument m_c/m_φ, corresponds to the first harmonic, and the right-hand branch corresponds to the second harmonic. The most stable are systems with identical partial frequencies $\omega_\varphi = \omega_c$, $\omega^2_\varphi = (k_2 + k_3)/m_\varphi$ and $\omega_c^2 = (k_2 + k_1)/m_c$, in which the quantity Ψ achieves the minimum value.

Despite the simplicity of the mechanical model considered, it very well represents the many properties of thermoinduced oscillations observed in practice: increased displacement amplitudes during oscillations; low vibroactivity of thermal systems in the form of single-side cooling of tubes closed from both ends with helium, and with a relatively high vibroactivity of systems in the form tubes with helium, closed at the warm end and open at the cold end, or U-shaped tubes with helium closed at the ends and cooled with helium ($k_3 = 0$, Fig. 2); a higher excitability of the first harmonic of the oscillations and a reduction of vibroactivity with increase of length of the warm section of the tube L_c for a constant length of the cold section $L\varphi$.

With regard to the excitation process of thermoacoustic self-oscillations itself, we associate it with breakdown of the rod analogy of acoustic oscillations during mass-exchange with the surrounding medium or with gas particles condensed specified points of the equipment. The model of relaxation oscillations is the most suitable for the correct description of these phenomena.

NOTATION

A, the coefficient of excitation; a, amplitude; c, coefficient of viscous drag; D, diameter; I, inertial number; k, coefficient of elasticity; L, length; m, mass; p, pressure; T, temperature; t_*, period of oscillations; t, time; V_*, velocity of sound; v, velocity; x, a coordinate; α, coefficient of radial heat transfer; γ, generalized (complex) frequency; δ, damping coefficient; ε, mass-exchange coefficient; Θ, temperature gradient; $\varkappa$, adiabatic coefficient; Λ, axial heat-transfer coefficient; ν, kinematic viscosity; ρ, density; φ, coefficient of negative friction; Ω, angular frequency; ω, partial frequency.

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